NAME (print): ____________________________

Math 203 Spring 2013—Exam 1

Instructor: J. Shapiro

Work carefully and neatly and remember that I cannot grade what I cannot read. You must show all relevant work in the appropriate space. You may receive no credit for a correct answer if there is insufficient supporting work. Notes, books and graphing or programable calculators are NOT ALLOWED.

[18pts] 1. Fill in the blanks with *always, sometimes, never* so that the following are correct statements.

(a) If A is a 3×3 matrix with three pivot points, then $Ax = \mathbf{0}$ ________ has a unique solution.

(b) If A is a 3×3 matrix with two pivot points, then $Ax = \mathbf{b}$ ________ has a solution.

(c) Let A be any $n \times m$ matrix. If u is a solution to $Ax = \mathbf{0}$ and w a solution to $Ax = \mathbf{b}$, then $u + w$ is ________ a solution to $Ax = \mathbf{b}$.

(d) If the vectors v_1, v_2, v_3 are linearly dependent vectors in $\mathbb{R}^4$ and v_4 is any vector in $\mathbb{R}^4$, then the set v_1, v_2, v_3, v_4 is ________ linearly dependent.

(e) If A is a 3×3 matrix such that $Ax = \mathbf{0}$ has many solution, then $Ax = \mathbf{b}$ ________ has a solution.

(f) Let A be a matrix such that the equation $Ax = \mathbf{0}$ has a unique solution. Then the columns of A are ________ linearly independent.

[16] 2. For each of the following augmented matrices, describe the solution set.

(a) $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 1 & 1 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 0 & 2 & -4 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$

[10] 3. For what value(s) of a is the following augmented matrix a consistent system?

$$\left[\begin{array}{rrr|r} 1 & 0 & 3 & 1 \\ 2 & 1 & 2 & -2 \\ 2 & -1 & a & 1 \end{array}\right]$$

[10] 4. Determine if the vector $\mathbf{b} = \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}$ is in the span of the set $\left\{ \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \right\}$

[10pts] 5. What should h be so that the vectors $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix}, \begin{pmatrix} 5 \\ 5 \\ h \end{pmatrix}$ are linearly dependent.

[10pts] 6. Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be a linear transformation such that

$T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$. Compute $T\begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$.

[10pts] 7. Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ be a linear transformation such that $T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a-b \\ 3a+b \\ a+b \end{pmatrix}$. Find the standard matrix of T.

8. Consider the following system of equations: $\begin{aligned} x_1 - 2x_2 &= 0 \\ 2x_2 - 4x_3 + 2x_4 + 2x_6 &= 0 \\ 3x_4 + 6x_5 &= 0 \end{aligned}$

[4pts] (a) Write down the associated augmented matrix for the above system.

[4pts] (b) List the free variables of the system.

[8pts] (c) Given that the matrix in part (a) reduces to $\left(\begin{array}{cccccc|c} 1 & 0 & -4 & 0 & -4 & 2 & 0 \\ 0 & 1 & -2 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 & 0 \end{array}\right)$, describe the solution set to the homogeneous system in parametric form.

[4pts] (d) Given that $v = [0, -1, 1, -1, 2, 1]$ is a solution to $Ax = b$, where A is the coefficient matrix of the original system and $b = \begin{pmatrix} 2 \\ -6 \\ 3 \end{pmatrix}$, use the previous part to write all solutions in parametric vector form to the matrix equation $Ax = b$.